Shot noise behavior of mesoscopic cavities in the presence of disorder

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Abstract. We analyze the effect of a random distribution of scatterers, satisfying the conditions for diffusive transport, inside a mesoscopic cavity. We show that in the presence of the scatterers the distribution of the transmission eigenvalues changes from that typical for an empty cavity to that characteristic of diffusive conductors and, consequently, the value of the shot noise suppression factor moves from 1/4 to 1/3.

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INTRODUCTION

Shot noise suppression in mesoscopic structures has been the topic of several theoretical and experimental studies in the last few years. In particular, it has been shown both with Random Matrix Theory (RMT) [1] and using a semiclassical approach [2] that in a diffusive wire (satisfying the condition $l \ll L \ll Nl$, where $l$ is the mean free path, $L$ is the length of the conductor and $N$ is the number of propagating modes [1]) the shot noise power spectral density is suppressed to $1/3$ of the Poissonian value $2e|I|$ (where $e$ is the elementary charge and $I$ is the average current), i.e. the so-called Fano factor is equal to $1/3$. For a mesoscopic cavity, delimited by narrow and identical constrictions, it has instead been theoretically proven by means of RMT [3] and with a semiclassical demonstration [4] that the shot noise power spectral density is suppressed down to $1/4$ of the full shot value. It has been shown that this result is valid regardless of the shape of the cavity (in particular also for rectangular cavities), due to the diffractive action of the constrictions [5, 6]. These theoretical results have been experimentally confirmed by Henny et al. [7] and Oberholzer et al. [8] for diffusive conductors and chaotic cavities, respectively.

In the recent literature, the effect on the distribution of the transmission eigenvalues, and therefore on the Fano factor, of the presence of disorder inside chaotic cavities has been investigated. In particular, Sukhorukov and Bulashenko [9] have theoretically modeled the effect on the distribution of the transmission eigenvalues of a weak short-range bulk disorder and Rotter et al. [10] have extended such a model including also the effect of scattering at the cavity walls.

We have instead studied, through large-scale numerical simulations based on the recursive Green’s function method, the effect that a random distribution of strong scatterers inside a cavity has on the distribution of the transmission eigenvalues and thus on the shot noise suppression. In the following we present our results, showing that, increasing
the strength of the disorder inside the cavity, we observe a crossover from the behavior typical of an empty cavity to that characteristic of diffusive conductors.

**NUMERICAL METHOD**

Our numerical simulations are based on the recursive Green’s function method [11, 12]. Using as a basis for representation real space in the longitudinal direction and mode space in the transverse direction, we find the diagonal Green’s function matrix of each of the sections of the device characterized by a longitudinally constant transverse potential, considering Dirichlet conditions at the boundaries. Then we recursively compose the Green’s function matrices of adjacent sections applying Dyson’s equation (where we introduce a perturbation potential which connects the sections) and we obtain the Green's function matrix of the overall structure, from which we derive the transmission matrix \( t \). From such a matrix we determine the conductance and the shot noise power spectral density using the Landauer-Büttiker formalism:

\[
G = \frac{2e^2}{h} \sum_{n,m} |t_{nm}|^2 = \frac{2e^2}{h} \sum_n T_n , \quad S_f = 4 \frac{e^3}{h} |V| \sum_n T_n (1 - T_n)
\]  

(1)

(\( h \) is Planck’s constant, the \( T_n \)'s are the eigenvalues of the matrix \( tt^\dagger \) and \( V \) is the externally applied voltage). The Fano factor is therefore computed as

\[
\gamma = \frac{\langle S_I \rangle}{\langle 2e|I| \rangle} = \frac{\langle S_I \rangle}{\langle 2e|V|G \rangle} = \frac{\sum n T_n (1 - T_n)}{\sum n T_n}
\]  

(2)

where we separately average the numerator and the denominator over a range of energies around the considered Fermi energy (in our simulations we have considered 100 energy values in a range of 40 \( \mu \)eV around the Fermi energy of 9.145 meV).

**RESULTS**

We have considered a 5 \( \mu \)m long and 8 \( \mu \)m wide rectangular cavity (a size corresponding to that adopted in experimental studies [8]), delimited by narrow input and output constrictions with various widths, aligned or with a relative vertical shift.

In the case of the empty cavity we have found a Fano factor equal to 1/4, as expected as a consequence of the diffraction at the narrow constrictions, which makes the cavity behave as a quasi-reservoir decoupling the two constrictions [5]. The distribution of the transmission eigenvalues \( T_n \) found for the empty cavity in the considered energy range is proportional to \( 1/\sqrt{T(1 - T)} \), as predicted by Random Matrix Theory [3]. This result is coherent with the value obtained for the Fano factor. Assuming the same distribution for each \( T_n \), we can write

\[
\gamma = \frac{\langle \sum_n T_n (1 - T_n) \rangle}{\langle \sum_n T_n \rangle} = \frac{\int_0^1 T(1 - T) \rho(T) dT}{\int_0^1 T \rho(T) dT} ,
\]  

(3)
FIGURE 1. The solid lines represent the distribution of the transmission eigenvalues obtained for a cavity with shifted 200 nm wide constrictions in the absence (a) and in the presence (b) of a random distribution of 1400 square scatterers (with a side of 40 nm and a height of 5 eV). The dashed curves are instead proportional to $1/\sqrt{T(1-T)}$ (a) and to $1/(T\sqrt{1-T})$ (b). The considered structures are sketched in the insets of the two panels.

and if $\rho(T)$ is proportional to $1/\sqrt{T(1-T)}$ this expression gives $\gamma = 1/4$. In Fig. 1(a) the solid line represents the distribution of the transmission eigenvalues for an empty rectangular cavity with 200 nm wide constrictions, vertically shifted with respect to each other. The dashed curve, instead, represents the theoretical result from Random Matrix Theory.

We have then included in the cavity a large number of randomly located square scatterers, with a density sufficient to lead to diffusive transport in a wire with the same width as the cavity (i.e. 8 $\mu$m wide). In particular, we have considered a random distribution of 1400 square scatterers (with a side of 40 nm and a height of 5 eV). At the considered Fermi energy of 9.145 meV, in the absence of the constrictions (and thus without the cavity) we obtain a Fano factor equal to 1/3 (corresponding to diffusive transport). Our numerical simulations have shown that also with the cavity the presence of randomly located scatterers leads to the same Fano factor of 1/3. Looking at the distribution of the transmission eigenvalues obtained for the cavity with the randomly located scatterers inside, we have found a curve proportional to $1/(T\sqrt{1-T})$, which is the distribution typical for diffusive transport [3]. We note that also in this case the result is coherent with the value of the Fano factor that we have found, since substituting in Eq. (3) a $\rho(T)$ proportional to $1/(T\sqrt{1-T})$ we obtain $\gamma = 1/3$. In Fig. 1(b) we show, with a solid line, the distribution of the transmission eigenvalues that we have found for the rectangular cavity containing a random distribution of 1400 square scatterers. The dashed curve is proportional to $1/(T\sqrt{1-T})$.

To gather a better understanding of the effect, we have performed numerical simulations varying the strength of the scatterers inside the cavity. Starting from scatterers with zero height (empty cavity) and gradually increasing such height, we have found a smooth transition from the behavior typical of a chaotic cavity to that typical of the diffusive regime: the Fano factor, starting from 1/4, approaches the 1/3 limit, while the distribution of the transmission eigenvalues gradually changes from a curve proportional to $1/\sqrt{T(1-T)}$ to a curve proportional to $1/(T\sqrt{1-T})$. For example, in Fig. 2 we show the results that we have obtained in the case of a cavity with aligned 400 nm...
FIGURE 2. (a) Fano factor for a cavity with aligned 400 nm wide constrictions and containing a random distribution of 1400 square scatterers (with a side of 40 nm), as a function of the height $V_0$ of the scatterers. (b) Distribution of transmission eigenvalues obtained for a scatterer height equal to 0 meV (solid line with solid circles), 6.25 meV (dotted line with solid squares) and 20 meV (dashed line with solid triangles). In the inset we report a sketch of the considered structure.

wide constrictions. In the left panel we report the Fano factor as a function of the value of the scatterer height, while in the right panel we represent the distribution of the transmission eigenvalues for a scatterer height equal to 0 meV (solid line with solid circles), 6.25 meV (dotted line with solid squares) and 20 meV (dashed line with solid triangles).

Our results clearly hint at the conclusion that the presence of an extended region with strong enough disorder has a dominant effect on the distribution of transmission eigenvalues, leading to that typical of diffusive transport, even if such a region is enclosed within a cavity that, by itself, would lead to a symmetric bimodal distribution. We believe that these findings are particularly significant, since, contrary to other results in the literature, which have explored the weak disorder case, they have been obtained treating a cavity of realistic size and with a disorder model that has been shown to provide a correct description of the diffusive limit [13].

REFERENCES