We present a review on some aspects of shot noise research, mainly focusing on our own work in the field, and on some conclusions that we can draw looking at the results obtained in the last few years. The activity on shot noise of one of us (M. M.) started during the preparation of his thesis, in 1986-1987, of which Lino was one of the advisors. That was the first exposure to the numerical simulation of noise phenomena, to shot noise, and to the effect on shot noise of the nanoscale size of devices. It was also an opportunity to realize how the application of numerical simulations could help gaining a better understanding of noise phenomena and, in particular, how it could help in the estimation of the limits of applicability of analytical models. This last issue will be the focal point of this paper, where we will try to summarize the results we have obtained on the extent to which realistic devices and nanostructures can exhibit the shot noise suppression factors predicted on the basis of realistic models.

INTRODUCTION

The interest in the numerical simulation of noise phenomena of one of us (M. M.) originated during the early trips to Modena in 1986, when he was introduced to the Monte Carlo method by Lino, one of the leading experts in the field. We applied the Monte Carlo method to the investigation of shot noise in a short conductor, into which carriers are injected, with a thermal distribution, from the contacts. It was relatively easy to show, on the basis of the numerical results, the transition from bulk thermal noise (if the length of the conductor was large compared to the mean free path between thermal scattering events) to shot noise (which became coincident with thermal noise in an equilibrium condition) when the conductor became shorter and shorter and therefore the transport regime became substantially ballistic. Such results were not published at the time, because, except for a few peculiar characteristics of the current autocorrelation function, they substantially corresponded to what had been found by Stanton and Wilkins [1] using a statistical analysis. This, however, provided an understanding of the potential of numerical simulation in the investigation of noise phenomena.

As a result, some time later, we decided to apply a numerical technique, based on the solution of the scattering problem through a ballistic nanostructure, to problems in which a suppression of shot noise as a result of carrier correlations was expected. Several seminal papers in the 80’s and early 90’s led to the realization that some “universal” suppression factors of the current noise power spectral density were to be observed in mesoscopic devices. A comprehensive overview of early results in this field is provided in the review by Blanter and Büttiker [2].

The ratio of the noise current power spectral density to that expected for full shot noise (for uncorrelated carriers) is defined “Fano factor”. In the case of uncorrelated carriers, the low-frequency shot noise power spectral density $S_I$ is given by Schottky’s theorem [3]:

$$S_I = 2qI,$$

where $q$ is the electron charge and $I$ is the average current through the device.

For example, it was determined, both on the basis of random matrix theory [4] and of a semiclassical technique [5], that the shot noise current power spectral density is suppressed by a factor 1/3 in disordered conductors in which transport can be considered to be diffusive, i.e. conductors whose length is much larger than the elastic mean free path (so that a significant number of elastic scattering events is expected to take place for each carrier crossing the device) and, at the same time, is much smaller than the product of the elastic mean free path times the number of propagating modes, approximately corresponding to the localization length (so that strong localization, which would lead to full shot noise, does not take place).

It was also shown [6] that, on the basis of a semiclassical model, a Fano factor of 1/3 would be reached also in a 1-dimensional structure with cascaded potential barriers, for a large enough number of barriers, regardless of the transparency of the barriers.

Another peculiar value of the Fano factor was determined by Jalabert et al. [7] for a structure consisting of a chaotic cavity, i.e. a conductor made up of a relatively large region (with a size of the order of a few micrometers) connected to external leads via two symmetric narrow constrictions, with quantized conductance. The Fano factor for such a structure was found to be 1/4 (as long as transport is ballistic and the constrictions are of the same width and much narrower than the cavity).

This result received an experimental confirmation in 2001 by Oberholzer et al. [8], who were able to measure suppression of shot noise by a factor 1/4 in a cavity defined (by means of depletion gates fabricated on the surface) in the 2-dimensional electron gas (2DEG) of a
GaAs/AlGaAs heterostructure.

Agam et al. [9] introduced the concept of a quantum to classical transition as the width of the constrictions is increased: when the constrictions are narrow, the dwell time in the cavity is long compared to the characteristic time for diffraction or Ehrenfest time, therefore we are in the “quantum” regime, in which trajectories can be split, thereby introducing an element of randomness and noise; when the constrictions are wide, the dwell time is short compared to the Ehrenfest time, and therefore the behavior is substantially classic and deterministic, with a complete suppression of noise.

Oberholzer et al. [10] performed further measurements on a structure analogous to that in Ref. [8], looking at the variation of the Fano factor as a function of the number of modes propagating in the constrictions and at the dependence of the Fano factor on an orthogonal magnetic field. They interpreted their results as a confirmation of the dependence of the Fano factor on the ratio of the dwell time to the Ehrenfest time formulated by Agam [9].

Another contribution on the suppression of shot noise in mesoscopic cavities was given by Silvestrov et al. [11], who argued that, as the system gets closer to the “classical” regime, fully transmitted or reflected noiseless states appear, which lead to reduced shot noise. An analytical expression was proposed for the behavior of the shot noise power spectral density as a function of the number of propagating modes and of the Lyapunov exponent.

In the following we will discuss how numerical simulations have been used to analyze the mentioned shot noise suppression phenomena, sometimes confirming the analytical results and in other cases pointing out their limits of validity.

**FANO FACTOR IN DISORDERED CONDUCTORS**

We have first focused on shot noise in 2-dimensional or 3-dimensional disordered conductors. We know, on the basis of the theory in Ref. [4] that, if the condition \( l_0 \ll L \ll Nl_0 \) (where \( l_0 \) is the mean free path, \( L \) is the length of the device, and \( N \) is the number of propagating modes) is satisfied, shot noise will be suppressed by a factor of 1/3. We performed a numerical simulation of shot noise [12], in which we considered a wire 200 nm wide and 800 nm long, containing randomly distributed square 12 x 12 nm hard-wall obstacles. From the transmission matrix (evaluated with the recursive Green’s function formalism [13, 14]) it was possible to compute the conductance (using the Landauer-Büttiker formula), the low-frequency shot noise power spectral density (following the approach delineated by Büttiker [15]), and therefore the Fano factor.

At the time we obtained the result reported in Fig. 1, where the Fano factor for different total numbers of impurities (from 30 to 180) is plotted as a function of the Fermi level. It is apparent that the shot noise suppression factor tends to an asymptotic value as the Fermi level (and therefore the number of propagating modes) is increased, but such an asymptotic value, although not too far from 1/3, does depend on the number of obstacles. From these results we concluded that further work was needed to assess the universality of the 1/3 suppression factor.

At about the same time Lino and coworkers opened new perspectives with Ref. [17], in which they found, with a Monte Carlo simulation coupled to a “dynamic” Poisson solver (i.e. taking into account the time dependence of the charge determining the electrostatic potential) that a suppression of shot noise down to 1/3 of the full shot value could be determined also by long-range Coulomb interaction, as long as the conductor is much longer than the elastic mean free path and of the Debye length.

Kolek et al. [18] published an interesting numerical analysis of shot noise suppression in disordered potentials, focusing on the weakly localized regime. They found that some analytical expressions for the transition from the diffusive to the weakly localized regime and from the diffusive to the ballistic regime were confirmed by numerical simulations. They also stated that reaching the fully diffusive regime in a numerical simulation would be difficult because of the large number of modes (in excess of 100) that would be needed.

Since at the time we were instead able to treat a few hundred propagating modes, we made an effort to determine the actual numerical conditions for which a diffusive regime could indeed be reached. By considering a number of different distributions of hard-wall impurities, we arrived at the conclusion [19] that a suppression factor 1/3 can be achieved with relative ease, as long as the inequalities \( l_0 \ll L \ll Nl_0 \) are verified by a factor of at least 3 or 10 in the lower or upper inequality, respectively, if hard wall scatterers are considered. It was in-
stead clear that the situation was different if obstacles of finite height (comparable or less than the Fermi energy) were assumed, as is the case in a realistic semiconductor device.

Indeed, while the suppression of shot noise down to 1/3 had been experimentally confirmed in the case of metallic wires [20], there was (and there still is) only one paper [21] attempting to provide experimental proof of such a suppression in a semiconductor nanostructure and it did not contain definitive evidence. In such a paper the authors presented results with a Fano factor varying between 0.2 and 0.4 as a function of the gate voltage, and therefore going through 1/3 only for a narrow interval of gate voltage values.

We then performed a simulation of shot noise in a realistic quantum wire, with a self consistent potential landscape [22] and with parameters tuned in such a way as to match experimental conductance data on an actual sample. Disorder, due to randomly located dopants, was added on top of the smooth potential obtained from the self-consistent solution of the Poisson and Schrödinger equations, using an analytical expression [23] for the inclusion of the screening by the 2DEG.

The behavior of the Fano factor, as a function of gate voltage, is reported in Fig. 2 for a wire 1 µm, 3 µm, and 5 µm long: it is apparent that it is not constant and it does cross the 1/3 value only for a specific gate voltage.

FIG. 2. Fano factor as a function of the gate voltage for a wire 1 µm (solid line), 3 µm (dashed line), and 5 µm (dotted line) long.

This is due to two facts: on the one hand, the potential perturbations due to the screened donors are not so strong to look like hard-wall scatterers for the electrons at the Fermi energy; on the other hand, even if they were hard-wall scatterers, it would be very difficult to achieve a fully diffusive transport, with a number of propagating modes within the limits (at most a few hundreds) of semiconductor-based mesoscopic devices, as will be discussed below.

We have recently performed a thorough exploration of the parameter space for model disordered conductors [24], considering different concentrations of scatterers and scatterer heights. In Fig. 3 we report the Fano factor as a function of the number of scatterers in a 4.9 µm wide and 8 µm long quantum wire, averaging over 41 energy values in a 80 µV window around a Fermi level assumed at 9 meV. Each scatterer is square (50 nm × 50 nm) and 15 meV tall. The Fano factor is close to 1/3 only for a narrow range of the number of square scatterers (between 1000 and 1600): this is a result of the fact that for lower scatterer densities the device length is not much greater than the mean free path any longer, while for higher scatterer densities the device length is not much smaller than the mean free path times the number of propagating modes (i.e. the localization length). Since it turns out that the inequalities need to be satisfied by a factor of about 10, the number of modes (which in the considered case is 320) must be very large (in the order of thousands) to have a relatively wide interval of the scatterer number within which diffusive transport is obtained, a situation that cannot be achieved but in macroscopic semiconductor devices.

This can be confirmed also with a more realistic disordered potential [24], created considering the electrostatic effect of ionized dopants located 40 nm above the 2DEG (in a GaAs/AlGaAs heterostructure): in Fig. 4 we present the Fano factor as a function of a scale factor applied to the disordered potential, for several values of the dopant density. We notice that for most of the reported densities the value of 1/3 is just crossed, only for a dopant concentration of the order of $10^{13}$ m$^{-2}$ there is a reasonably wide interval within which the diffusive result is obtained. This corresponds, however, to very large, isolated bumps that do not correspond to the potential that is usually expected in semiconductor nanostructures as a result of dopants.

Thus, from these calculations and from the existing experimental data, we can conclude that a fully diffusive behavior is not expected to be common in semiconductor-based nanodevices.
FIG. 4. Fano factor as a function of a scale factor applied to the disordered potential, for a dopant density equal to: $2 \times 10^{15}$ (thick solid line), $1.1 \times 10^{15}$ (thin solid line), $1.1 \times 10^{14}$ (thick dashed line), $1.1 \times 10^{13}$ (thin dashed line) and $1.1 \times 10^{12}$ (dotted line).

FANO FACTOR IN MESOSCOPIC CAVITIES

In the Introduction we have mentioned the main papers containing experimental data or analytical models concerning the suppression down to $1/4$ of shot noise in mesoscopic cavities. Also in this case numerical techniques can help getting a better understanding of the phenomenon.

We performed one of the first numerical calculations [25] of shot noise suppression in mesoscopic cavities, considering a rectangular cavity (i.e. with a classically regular, non-chaotic shape), obtaining a clear value of $1/4$ for the Fano factor, as shown in Fig. 5, where we report the shot noise suppression factor as a function of the Fermi energy for a hard-wall rectangular cavity $5 \, \mu m$ wide and $7.75 \, \mu m$ long, with symmetric $1 \, \mu m$ wide constrictions (the GaAs effective mass is assumed throughout all of our calculations). This was not competently expected on the basis of theories existing at the time, which focused on the “chaotic” nature of the cavities and in most cases considered either generic classically chaotic shapes or, specifically, a Bunimovich billiard (i.e. a stadium). Based on most of the theoretical papers (such as Ref. [9]) the diffraction at the origin of the “quantum” behavior of the system, leading to zero-temperature shot noise, takes place everywhere in the cavity and in Ref. [10] it is even stated that, when motion of the electrons is limited by magnetic field to an “annulus” forming a smaller cavity, the smaller cavity leads to shot noise because the dynamics is still chaotic due to the presence of “impurities or irregularities in the geometry of the cavity”. This is absolutely not needed, because the relevant diffraction (and the ensuing chaotic dynamics) is only at the constrictions and results from the change of width from the cavity to the constriction itself.

Nazmitdinov et al. [26] focused on cavities with large openings, finding that in this particular case the shape of the cavity and the orientation of the leads appeared to play a role on the Fano factor.

In an interesting contribution by Aigner et al. [27] it was instead shown that, in the condition of narrow openings, the shape of the cavity does not play a significant role on the Fano factor, thus confirming our interpretation about relevant diffraction occurring only at the constrictions.

Prompted by the results in Ref. [10], we performed a numerical simulation [16] of both parts of the experiment: i) the evaluation of the Fano factor as a function of the number of modes propagating through the constrictions and ii) the evaluation of the Fano factor as a function of the orthogonal magnetic field for a given width of the constrictions. We considered a model hard-wall cavity in order to speed up the calculations, since in a few test calculations we had not found significant differences with respect to a classically chaotic shape, such as a stadium.

In Fig. 6 we show the Fano factor as a function of the number of modes propagating in the cavity: the solid triangles correspond to the experimental values of Ref. [10], while the empty circles correspond to the outcome of the numerical simulation. The length of the considered rectangular cavity is $5 \, \mu m$, as the distance between the quantum point contacts representing the constrictions, while the width is $8 \, \mu m$, on the assumption (later confirmed from other sources) that the sample is the same as the one in Ref. [8].

The agreement between the simulation and the experimental data is surprisingly good, considering that there is no fitting parameter. This further confirms that what really matters is the width of the constriction with respect to the width of the main cavity. We also notice that the agreement with the experiment of this numerical model is much better than the one of the analytical expression from Silvestrov et al. [11], who state that a Fano factor of $1/4$ should be observed until the num-
number of modes $N$ propagating in each constriction satisfies the inequality $N \leq \sqrt{k_F L}$, where $k_F$ is the wave vector at the Fermi energy and $L$ is the distance between the two constrictions. In Ref. [8] the Fermi energy of the 2DEG is given as 106 K, which, in more common units, corresponds to 9.136 eV or $1.464 \times 10^{-21}$ J. Thus the Fermi wave vector $k_F$ will be given by $k_F = \sqrt{2mE_F}/\hbar = 1.266737 \times 10^8$ m$^{-1}$, where $m$ is the effective mass of Gallium Arsenide, $E_F$ the Fermi energy, and $\hbar$ is the reduced Planck constant. Thus $\sqrt{k_F L} = 25.1668$, which means that, according to the theory of Ref. [11], the Fano factor should be 1/4 for all the experimental points with $B = 0$ of Ref. [10] except for the one corresponding to $N = 40$, in clear disagreement with the measured data.

For the case of $B \neq 0$ our numerical simulation, performed with a slightly modified implementation of the method proposed by Tamura and Ando [28] and a recursive scattering matrix technique allowed [16] to understand the actual origin of the increasing shot noise suppression with increasing magnetic field. Contrary to the explanation in Ref. [10], where the further reduction of shot noise is associated with the interplay between the cyclotron radius and the cavity dimensions, our data suggest a much more direct explanation: when the cyclotron diameter is large compared to the width of the leads, diffraction at the leads is about the same as in the absence of magnetic field; as the cyclotron diameter becomes comparable with the width of the leads, diffraction is reduced, which, in turn, determines a further suppression of shot noise, until the cyclotron diameter becomes less than the width of the leads, and edge states can travel unimpeded across the cavity, without any diffraction and therefore without any shot noise. In Fig. 7 we plot the Fano factor as a function of the ratio of the cyclotron diameter to the width of the leads, for different values of the constriction width: it is apparent that a similar behavior is obtained for each width, confirming that the relevant parameter is just the mentioned ratio.

**SHOT NOISE IN THE PRESENCE OF 1-DIMENSIONAL DISORDER**

As we have discussed in the introduction, semiclassical approaches [6, 29] to the calculation of the Fano factor in a structure with a series of cascaded barriers yield a value of 1/3 as the number of barriers is increased, regardless of their transparency. This, however, is not confirmed by the only existing experimental test [30].

We have performed a quantum calculation of shot noise suppression in a series of barriers [31], finding values that strongly depend on barrier transparency, without any clear trend towards 1/3. In Fig. 8 we report the Fano factor as a function of the number of barriers for various transparencies. We have performed averaging over the interbarrier distances, which have in any case to be chosen in such a way to be different between each pair of barriers, to avoid resonance effects. The reason of the discrepancy between the semiclassical and the quantum model is that localization appears in the latter, because there is no mode mixing. The only way to approach the diffusive limit is to introduce some source of mode mixing, such as an orthogonal magnetic field. In the absence of magnetic field we do not expect diffusive transport to be observable in practical samples with 1-D disorder.

**CONCLUSION**

We have presented the results obtained applying the tool of numerical simulation to the investigation of shot noise suppression in disordered conductors and in mesoscopic cavities. We have been able to determine that a Fano factor of 1/3 is not so ubiquitous as might be expected from most of the literature. In particular, in
semiconductor nanostructures the condition of fully diffusive transport can be reached only in rather peculiar conditions, thus explaining why a dependence of the Fano factor on gate voltage had been observed in experiments. Furthermore, we have found that 1-dimensional disorder does not lead to diffusive behavior, due to the lack of mode mixing and therefore to the substantial correspondence between the mean free path and the localization length. Only in the presence of a magnetic field some mode mixing is created and this leads to the shot noise suppression factor approaching 1/3 for a few values of the barrier transparency. Thus the shot noise suppression factor 1/3 is not too “universal,” at least for nanodevices, because it can be observed only in very particular situations.

In the case of mesoscopic cavities, numerical simulations have clarified that the shape of the cavity is influential and that it does not have to be chaotic to achieve the suppression factor 1/4, because the only “chaoticity” rests in the diffraction that takes place at the constrictions. In this sense the 1/4 shot noise suppression factor is more “universal” than expected from most of the literature, since it is not restricted to cavities with chaotic shapes, although nonnegligible deviations are observed in the presence of asymmetries in the main body of the cavities.

Furthermore, it has been possible to gain a better understanding of the role of magnetic field in suppressing noise in a cavity, demonstrating that the only relevant parameter is the ratio of the cyclotron diameter to the width of the leads.


FIG. 8. Fano factor as a function of the number of barriers for a transparency equal to 0.1 (dotted line), 0.5 (dashed line) and 0.9 (solid line); these results have been obtained adopting a simplified model in which the barrier transparency is independent of the orthogonal wave vector of the impinging particle.