Numerical investigation of noise and transport properties of multiple mesoscopic cavities

Paolo Marconcini and Massimo Macucci

Dipartimento di Ingegneria dell’Informazione, Università di Pisa
Via Caruso 16, I-56122 Pisa, Italy

ABSTRACT

We study, using numerical simulations, the transport and noise properties both of a series of barriers and of cascaded constrictions, comparing our results with the conclusions of previous analyses. In particular, we point out the differences existing between the case in which the barriers or the constrictions are evenly spaced and the case in which they are randomly spaced, proposing possible explanations for the observed phenomena.

Keywords: tunnel barriers, chaotic cavities, Fano factor, resistance

1. INTRODUCTION

Mesoscopic devices containing relatively large areas of two-dimensional electron gas (2DEG) separated by narrow constrictions (defining so called chaotic cavities) or by potential barriers (Fig. 1) can be fabricated in a straightforward way with current GaAs/AlGaAs heterostructure technology.\textsuperscript{1} Both potential barriers and constrictions can be created with negatively biased metallic gates that are lithographically defined across a mesa delimiting a 2DEG in the transverse direction or with controlled etching.

The conductance and shot noise behavior of structures with cascaded barriers and with cascaded constrictions (delimiting mesoscopic cavities) has raised significant interest in recent years. In particular, using semi-classical techniques and some approximations, it has been shown that increasing the number of cascaded elements the

![Figure 1. Series of barriers (a) and series of constrictions (b), delimiting rectangular mesoscopic cavities.](image-url)


Proc. of SPIE Vol. 6600 660009-1
Fano factor should asymptotically reach the value 1/3 both in the case of barriers\(^2\) and in that of constrictions,\(^3\) as for diffusive transport.

Our interest in this problem has been prompted by the observation that quantum models for simple structures of this type (in particular with multiple constrictions\(^4\)) have yielded results that seem to be in contrast with the semiclassical predictions. This is particularly surprising when we consider that, instead, for diffusive conductors with randomly distributed scatterers the same quantum models have given results\(^5\) in perfect agreement with models based on random matrix theory\(^7\) and with the experiments,\(^8\) i.e. a Fano factor of exactly 1/3.

2. NUMERICAL METHOD

Our calculations have been performed with the recursive Green’s function technique, partitioning each nanostructure of interest into sections, each with a longitudinally constant transverse potential. For each section we assume Dirichlet boundary conditions at the boundaries and compute the associated Green’s function. Using as a basis for representation real space in the longitudinal direction and mode space in the transverse direction, the Green’s function for each section consists of a diagonal matrix whose elements can be evaluated from analytical expressions.\(^9\) The Green’s function matrices are then composed recursively, opening up adjacent sections with a proper perturbation \(V\) and applying Dyson’s equation, thus obtaining the Green’s function matrix for the whole device. From such a Green’s function, the transmission matrix \(t\) of the structure can be straightforwardly derived\(^6\) and from it the conductance and the shot noise power spectral density at low frequency can be determined using the Landauer-Büttiker formalism:

\[
G = \frac{2e^2}{h} \sum_{n,m} |t_{nm}|^2 = \frac{2e^2}{h} \sum_n w_n \quad , \quad S_I = \frac{4e^2}{h} |V| \sum_n w_n (1 - w_n)
\]  

(1)

(where \(e\) is the elementary charge, \(h\) is Planck’s constant, the \(w_n\)’s are the eigenvalues of the matrix \(tt^\dagger\) and \(V\) is the externally applied voltage). We have then computed the Fano factor (the ratio \(S_I\) to the full shot power spectral density \(2eI\), where \(I\) is the average current) as

\[
\gamma = \frac{\sum_n w_n (1 - w_n)}{\sum_n w_n} .
\]  

(2)

Due to the presence of fluctuations deriving from quantum interference effects, we average the results obtained over a range of energies around the considered Fermi energy (in the case of the Fano factor the average has to be performed separately on the numerator and denominator of the corresponding expression, as it would happen in an actual measurement procedure).

We point out that the calculation of the transmission matrix for cascaded barriers is particularly simple, since there is no mode-mixing at the interfaces, and therefore the problem is equivalent to a collection of one-dimensional problems, each for the corresponding value of the longitudinal component of the wave vector.

3. SERIES OF BARRIERS

We have first considered a structure 8 \(\mu\)m wide, consisting of a series of 4 nm thick and 25 meV high barriers, analyzing its behavior in terms of conductance and shot noise suppression over 200 energy values in a 0.12 meV wide range of energies (corresponding to a bias of about \(9kT/\epsilon\) at 150 mK) around 9.08 meV, a typical Fermi energy for the 2-dimensional electron gas (2DEG) in a GaAs/AlGaAs heterostructure. The series of barriers defines a cascade of closed cavities, each with a discrete number of allowed states; therefore resonances play a major role in transport through this structure.

The behavior of the structure is different for a series of equally spaced barriers (in our simulation we have considered a separation of 5 \(\mu\)m) and for a series of barriers with nonuniform spacing (for our computations we have considered the following distances between the barriers: 5.25, 4.5, 4.25, 5.5, 5.125, 4.75, 5.5 and 4.625 \(\mu\)m). Indeed, in the first case we are in the presence of identical cavities, characterized by stationary states with the same energies, therefore the overall structure has a significant number of resonant energies. In particular this can be seen from the presence of numerous peaks in the contribution of each mode to the overall
conductance as a function of energy, that, increasing the number of barriers, coalesce into energy bands, in which the considered mode is allowed to pass through the device (as it is known to happen in a periodic structure, using the Kronig-Penney model). Instead, in the case of barriers located at different relative distances, the resonances partially disappear, since the cavities are in this case characterized by different allowed energies. This fact, which is confirmed by the reduced number of peaks in the contribution of each mode to the overall conductance as a function of energy, leads to a lower value of the conductance for the overall structure.

In particular, as can be seen from Fig. 2, where we show the resistance of a series of barriers as a function of the number of cavities that they define, the overall resistance is approximately equal to that of a single cavity if the barriers are evenly spaced, while the resistance of the structure increases proportionally to the number of cavities if the barriers are separated by uneven distances.

In Fig. 3, instead, we report the results obtained for the same structures in terms of the Fano factor, as a function of the number of cavities. The Fano factor approaches 0.28 for equally spaced barriers, while it settles around a larger value (approximately 0.45) for unevenly spaced barriers. We notice that both results differ from those obtained with a semi-classical procedure, according to which the Fano factor of a series of barriers should approach 1/3 as the number of cascaded barriers is increased.

In Fig. 4, we report the distribution of the transmission eigenvalues in the structure (the $w_n$’s of Eq. 1), which shows that in the case of evenly spaced barriers we have a distribution with peaks in correspondence of 0 and 1, while in the case of non-evenly spaced barriers the peak at 1 disappears, due to the strong suppression of the resonance phenomena, and the distribution has a monotonic behavior.

4. SERIES OF CONSTRICITIONS

We have then studied the conductance and noise for a series of hard-wall constrictions defining rectangular mesoscopic cavities. In a semiclassical model, if each cavity behaves as a quasi-reservoir, we could consider it as a node joining adjacent constrictions, which we could represent as noisy resistors with transmission eigenvalues equal to 0 or 1. In this hypothesis, one obtains that the resistance of the overall structure should be equal to the series of the resistances of the constrictions, while the Fano factor should approach 1/3 as the number of constrictions increases.

We notice that a cavity behaves as a quasi-reservoir if inside it all the states at the same energy have the same occupancy. The presence of a discontinuity in the potential in correspondence with the delimiting constrictions generates mode-mixing effects which allow electrons to explore all the transverse modes inside the cavity and
Figure 3. Fano factor, as a function of the number of cavities, for a series of evenly spaced barriers (solid squares) and for a series of unevenly spaced barriers (solid dots).

Figure 4. Distribution of the transmission eigenvalues in a series of 10 evenly spaced barriers (solid curve) and in a series of 10 unevenly spaced barriers (dashed curve).
Figure 5. Resistance of a series of evenly spaced identical constrictions as a function of the number of cavities defined by the constrictions.

Figure 6. Fano factor of a series of evenly spaced identical constrictions as a function of the number of cavities defined by the constrictions.

to satisfy the quasi-reservoir hypothesis (also in the case of an integrable, for example rectangular, shape of the cavity).

We have considered evenly and unevenly spaced constrictions, observing different transport and noise behaviors. In Fig. 5 we show the resistance and in Fig. 6 the Fano factor for a series of 200 nm wide constrictions, as a function of the number of cavities they define. Constrictions are evenly spaced (5 µm apart), and there is a clear disagreement with respect to the results of the semi-classical analytical model: resistances do not add and the Fano factor does not approach 1/3 as the number of cavities is increased. On the contrary, we see that the overall resistance doubles at most with respect to that of a single cavity, while the Fano factor, starting from 1/4 (the result valid for a single mesoscopic cavity with narrow and symmetrical constrictions), slightly decreases (instead of increasing towards 1/3).

Also in this case the results change if we consider a series of unevenly spaced constrictions (i.e. a series of cavities with different length). We report results for unevenly spaced cavities (5.05, 4.95, 5.1, 4.9, 5.15, 4.85 µm...
Figure 7. Resistance as a function of the number of cavities for a series of unevenly spaced constrictions (solid squares) and for a series of constrictions with different vertical position (solid dots).

Figure 8. Fano factor as a function of the number of cavities for a series of unevenly spaced constrictions (solid squares) and for a series of constrictions with different vertical position (solid dots).

long): the resistance of the structure (Fig. 7, solid squares) increases linearly or more than linearly with the number of cavities, while the Fano factor (Fig. 8, solid squares) increases towards a value significantly larger than 1/3. Similar results have been obtained in the case of cascaded cavities differing for the vertical position of the constrictions rather than for their length. This, too, can be seen in Figs. 7 and 8, where we report (with solid dots) the conductance and the Fano factor for a series of cavities with the same length but different vertical positions of the constrictions (shifted by 1200, -1200, 1800, -1800, 2400, -2400 nm), as a function of the number of cavities.

We observe that in the case of equally spaced, aligned constrictions, the overall behavior is quite similar to that of just a pair of constrictions forming a single cavity, with a resistance close to the series of the resistances of two constrictions and a Fano factor equal to 1/4 (for symmetric widths). Nevertheless, even extremely small differences, in the length or in the vertical position of the constrictions between the cascaded cavities, destroy
such an effect.

We propose an explanation for this phenomenon, based on the structure of the modes propagating inside a cavity delimited by narrow constrictions. In this case the longitudinal component of the modes, being very close to that of the corresponding closed cavity, will have nodes that periodically repeat in the propagation direction. These nodes coincide, for a series of exactly identical cavities, with the positions of the constrictions (in particular the intermediate ones) and thus reduce their effect on conductance and noise. Therefore the transport and noise behavior of a series of equally spaced constrictions is not too different from that of just a couple of constrictions, while this effect is clearly not valid for a series of unevenly spaced constrictions. On the other hand, the presence of a beaming effect, i.e., of a partial direct transmission between the constrictions, which clearly increases in the case of aligned constrictions, could explain the different behavior that we have observed between the cases of a series of constrictions with the same vertical positions and a series of vertically shifted constrictions. Indeed, without constriction alignment, the conductance decreases, while the Fano factor increases.

As a further annotation, we have also found that, reducing the width of an intermediate constriction, we observe a marked reduction of the conductance through the structure (as a result of the reduction of the number of propagating modes), while this does not have a large effect on the Fano factor. For example, considering simply the series of three constrictions which identify two cascaded 5μm long cavities, we have found in the case of identical 200 nm wide constrictions a resistance equal to 4239 Ω and a Fano factor 0.243; enlarging the central constriction to 400 nm we have found a resistance of 3868 Ω and a Fano factor 0.278; instead considering a 100 nm wide central constriction we have obtained a resistance of 7159 Ω and a Fano factor 0.249.

5. CONCLUSION

We have presented a numerical analysis of the shot noise suppression and of the resistance of mesoscopic channels containing cascaded barriers or constrictions. Our quantum mechanical model provides results that differ significantly from those of semiclassical calculations that can be found in the literature; in particular we do not notice an asymptotic behavior towards the diffusive limit as the number of barriers is increased. This leads us to the conclusion that there must be some fundamental difference between the description of this type of system with a quantum model in the Landauer-Büttiker scattering picture and the description obtained from a classical representation modified with the inclusion of the Pauli principle.

It would be extremely interesting to obtain detailed experimental results on these structures, in order to be able to assess which approach provides a better description of physical reality.

REFERENCES